An Integrated Environment for the Design, Simulation and Testing of Multidimensional Digital Filters

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***Abstract* - An environment for the design and simulation of multidimensional (M-D) digital filters is described, consisting of a color real-time video acquisition and display system and an M-D filter analysis and simulation facility, capable of performing comprehensive frequency-domain analysis and precisely-controlled spatiotemporal-domain simulations from SFG descriptions of M-D filters.**

I. Introduction

Over the past two decades, there has been increasing interest in the design of multidimensional (M-D) digital filters [1]. Among the many proposed design methods are those for the design of high-quality 2-D recursive filters having circularly-symmetric or fan-shaped responses [2]. Other techniques include the design of 3-D recursive filters from simple first- and second-order 3-D inductance-resistance networks for the enhancement of so-called *linear-trajectory* and *spatially-planar* spatiotemporal images [3]. M-D Discrete Fourier transform / linear difference equation (DFT/LDE) mixed-domain methods have also been recently reported [4]. Although there are many high-quality M-D filter designs available, the lack of a powerful M-D simulation facility has prevented the full simulation of many designs.

In this contribution, we describe an environment for the full design, simulation and testing of M-D digital filters using a Multi-Dimensional Network Analysis Program (*MDNAP*) in conjunction with a real-time color video image acquisition system. To our knowledge, it is the first such environment for this purpose.

II. Overview of Mdnap

MDNAP is a comprehensive CAD tool for discrete-domain 1,2 or 3-dimensional filters and in a limited capacity for continuous-domain filters. Based on the Signal-Flow Graph (SFG) description of the filter, *MDNAP* is capable of performing frequency-domain and sensitivity analysis for both discrete-domain and continuous-domain filters and spatiotemporal-domain analysis for discrete-domain filters. Included, for frequency-domain analysis, are functions which obtain the state-space difference equation, the transfer function and calculate the frequency response.

The sensitivity analysis formulates the first-order sensitivity function and calculates the first-order sensitivity of the frequency response function with respect to any multiplier coefficient in the SFG.

Spatiotemporal-domain analysis of discrete-domain filters formulates functions that provide scheduling information for sequential and parallel computations and information on memory requirements.

In addition to a SFG description of the filter, *MDNAP* offers a means for incorporating fixed-point precision of filter coefficients and node variables.

MDNAP allows multiplier coefficients to be either numeric or symbolic. When a command is executed, symbolic coefficients can be either matched with user-defined variables and evaluated, or preserved as symbols in the state-space difference equation, transfer function or sensitivity function. Both, frequency response and sensitivity can be evaluated either by forming and evaluating the expression, or numerically, by solving the state-space difference equation at each frequency point using LU decomposition.

As an option, 3-D frequency response and sensitivity functions can be rendered as shaded volumes to provide meaningful visual representation.

MDNAP's spatiotemporal-domain analysis can handle arbitrary orthogonal and hyperdiagonal (or diagonal) processing orders, and offers control over boundary conditions. *MDNAP* allows the use of both advance and delay operators, subject only to the requirement that the resulting SFG be realizable with the particular processing order selected. Many more command options are available, which offer precise control over *MDNAP* operations, but are omitted here for brevity.

MDNAP normally runs in an interactive mode, where the user enters commands which are immediately executed. The user can enter and edit the SFG and perform the analysis as many times as needed, and then save the work in a readable format. Alternatively, MDNAP can run in the background by executing commands from files.

MDNAP source code is written in *C* and, to date, has been compiled for *SUN SPARCstation*s, *NeXTstation*s and DOS-based Intel 80386/80486 PCs.

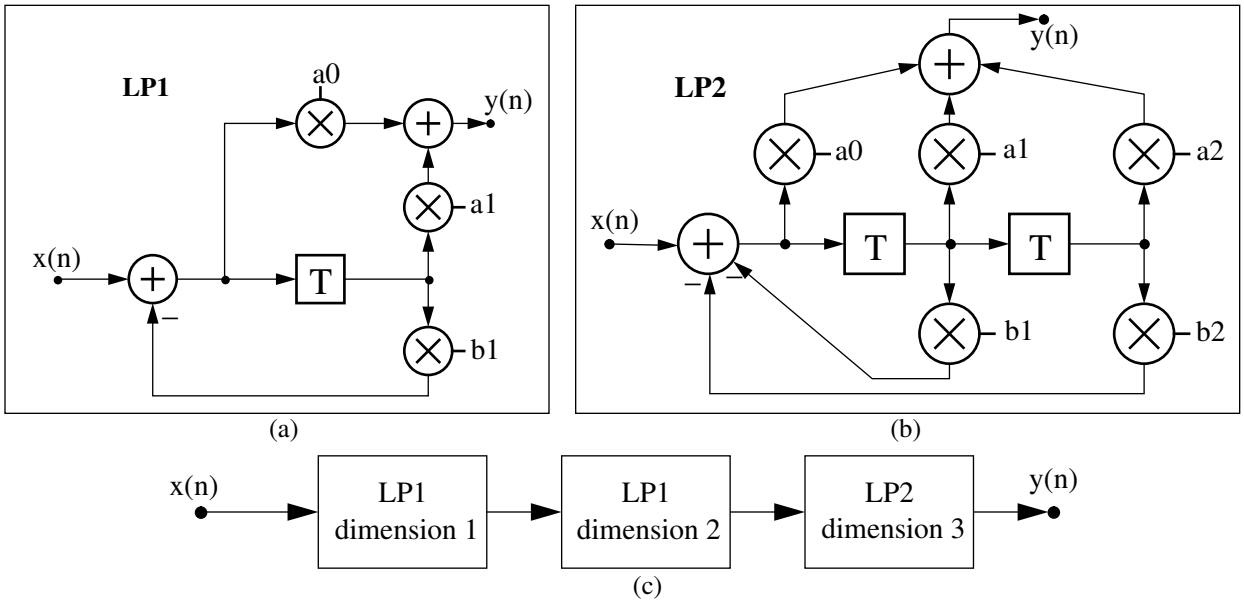

Figure 1. (a) 1st-Order Lowpass Section (b) 2nd-Order Lowpass Section (c) 3-D Lowpass Filter

III. An Example of a 3-D SFG Description

A 3-D separable lowpass filter, composed of cascaded 1-D first-order direct-form lowpass sections in the spatial dimensions (1 and 2) and a 1-D second-order direct-form lowpass section in the temporal dimension (3), is shown in Figure 1.

The structure of the filter is entered into *MDNAP* as a netlist that describes the SFG. As a convenience, macros can be utilized for repeating structures. The following *MDNAP* commands describe the SFG in Figure 1(c):

```
# 1st order lowpass macro definition
macro ord1lp # delay in out a0 a1 b1
 $1 $2 %1
 mult %1 %2 $6
 mult %2 $2 -1
 mult $2 $3 $4
 mult %1 $3 $5
.

# 2nd order lowpass macro definition
macro ord2lp #delay in out a0 a1 a2 b1 b2
 $1 $2 %1
 mult %1 %3 $7
 mult %3 $2 -1
 $1 %1 %2
 mult %2 %4 $8
 mult %4 $2 -1
 mult $2 $3 $4
 mult %1 $3 $5
 mult %2 $3 $6
.

# filter structure
in a
ord1lp z1 a b a0ord1 a1ord1 b1ord1
ord1lp z2 b c a0ord1 a1ord1 b1ord1
ord2lp z3 c d a0 a1 a2 b1 b2
out d

# LP1 coefficients (a = 1.7)
set a0ord1 0.6296296296296296
set a1ord1 a0ord1
set b1ord1 0.2592592592592593
# LP2 coefficients (a = 0.03)
set a0 0.000862625535089243
set a1 0.001725251070178486
set a2 a0
set b1 -1.915220382461473
set b2 0.91867088460183
```

If required, coefficient precision information can be entered in the above description. Node precision information can also be specified, before proceeding to analyze the filter.

IV. Symbolic M-D Transfer Functions and Frequency Response

Formulation of the state matrix difference equation is the basis for all *MDNAP* frequency-domain analysis functions. The equation is based on Roesser's model [5] and takes the form:

$$\begin{bmatrix} \underline{x}^h(i \pm 1, j, k) \\ \underline{x}^v(i, j \pm 1, k) \\ \underline{x}^t(i, j, k \pm 1) \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \underline{x}^h(i, j, k) \\ \underline{x}^v(i, j, k) \\ \underline{x}^t(i, j, k) \end{bmatrix} + \boldsymbol{B}u(i, j, k) \qquad (1)$$

$$y(i, j, k) = \boldsymbol{C} \begin{bmatrix} \underline{x}^h(i, j, k) \\ \underline{x}^v(i, j, k) \\ \underline{x}^t(i, j, k) \end{bmatrix} + du(i, j, k) \qquad (2)$$

In order to formulate matrices $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$ and scalar d, it is necessary to be able to calculate the overall path transmittance between any two SFG nodes [6]. To obtain the overall path transmittance between two SFG nodes, all paths between the two nodes must be found; transmittances of paths which are in series are multiplied and transmittances of parallel paths are added. Matrix $\boldsymbol{A}$ and vector $\boldsymbol{B}$ are formed by traversing the SFG against the direction of the transmittances, searching out all paths between the input to each delay/advance branch, and the output of all delay/advance branches and the filter input. Each overall path transmittance forms one entry in the matrix. Similarly, vector $\boldsymbol{C}$ and the scalar d are obtained by finding the overall path transmittances between the filter output, and the output of a delay/advance branch and the filter input.

Taking the 3-D z-transform of Equations (1) and (2), combining, rearranging and assuming zero initial conditions gives the expression for the transfer function,

$$H(z_1, z_2, z_3) = \boldsymbol{C}\left[\begin{bmatrix} z_1^{\pm 1}\boldsymbol{I} & 0 & 0 \\ 0 & z_2^{\pm 1}\boldsymbol{I} & 0 \\ 0 & 0 & z_3^{\pm 1}\boldsymbol{I} \end{bmatrix} - \boldsymbol{A}\right]^{-1} \boldsymbol{B} + d \qquad (3)$$

where $\boldsymbol{I}$ is the identity matrix.

The *algebraic representation* of the transfer function of Figure 1 is listed by *MDNAP* as follows:

```
numerator:
(a0*a0ord1^2)*z1*z2*z3^2
(a1*a0ord1^2)*z1*z2*z3
(a2*a0ord1^2)*z1*z2
(a0*a1ord1*a0ord1)*z1*z3^2
(a1*a1ord1*a0ord1)*z1*z3
(a2*a1ord1*a0ord1)*z1
(a0*a0ord1*a1ord1)*z2*z3^2
(a1*a0ord1*a1ord1)*z2*z3
(a2*a0ord1*a1ord1)*z2
(a0*a1ord1^2)*z3^2
(a1*a1ord1^2)*z3
(a2*a1ord1^2)
denominator:
(1)*z1*z2*z3^2
(b1)*z1*z2*z3
(b2)*z1*z2
(b1ord1)*z1*z3^2
(b1*b1ord1)*z1*z3
(b2*b1ord1)*z1
(b1ord1)*z2*z3^2
(b1*b1ord1)*z2*z3
(b2*b1ord1)*z2
(b1ord1^2)*z3^2
(b1*b1ord1^2)*z3
(b2*b1ord1^2)
```

With $z_i=e^{j\omega_i}$, the frequency response of the filter can be obtained by either directly evaluating the above transfer function or by numerically solving equation (3). By default, *MDNAP* employs the transfer function, but provides the option to use the numerical method, which allows for the evaluation of the frequency response for very large structures, for which the CPU time required to calculate the transfer function is prohibitive. In the case of numerical solution, LU decomposition with implicit pivoting is used to perform matrix inversions.

4.1 Visualization of 3-D Frequency Responses

The results of a frequency response calculation can be saved in a file for plotting with most general-purpose plotting utilities. The frequency response of a 3-D filter can also be displayed using *surf*, a 3-D volume rendering algorithm that has been incorporated into *MDNAP*'s frequency response functions. *Surf* renders 3-D frequency response or sensitivity functions, either magnitude or phase, as shaded, solid, contour volumes and is designed to provide a meaningful visual interpretation to the 3-D functions. Multiple contours can be rendered and optional clipping planes may be specified to cut away volumes and expose interior volumes. *Surf* employs conventional diffuse-reflection and specular-reflection illumination models [7] and allows arbitrary placement of viewing and light source positions. The magnitude frequency response of the filter in Figure 1, calculated by *MDNAP* and displayed with *surf*, is shown in Figure 2.

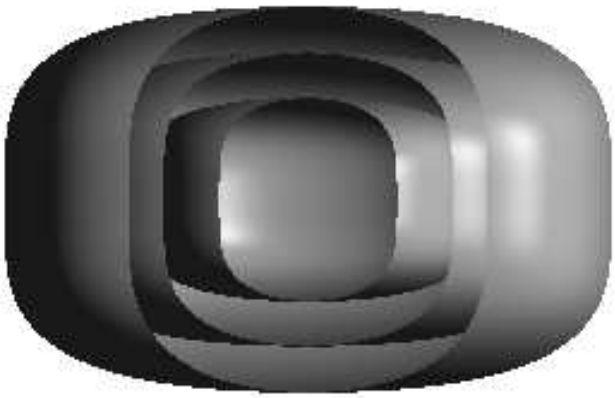

Figure 2. 3-D Lowpass Frequency Response

4.2 Sensitivity Functions

MDNAP can also symbolically generate the first-order sensitivity function of the transfer function to particular coefficients. Algebraic first order gain and phase sensitivities can be evaluated from the sensitivity function or they can be determined numerically by perturbing the coefficient of interest by Δx_i and numerically solving the two resulting matrix equations (the perturbed and the original) at a particular frequency point. The sensitivity expression is displayed by *MDNAP* in a form similar to that of the transfer function expression, and the calculation of sensitivity can be saved in a file, for plotting with other utilities, or displayed using *surf*.

V. Spatiotemporal-Domain Analysis

MDNAP's spatiotemporal-domain analysis functions are capable of simulating an arbitrary orthogonal processing order as well as 3-D hyperdiagonal (and 2-D diagonal) maximum-concurrency processing. Both delay and advance operators are supported, provided that the resulting SFG is realizable with the selected processing order. *MDNAP* can fill the data boundary region (i.e. the region which is outside of the data volume but is reachable by the processing mask) with any constant, or by mirroring the adjacent data points within the data volume. Coefficient and node quantization and overflow effects can be simulated with *MDNAP* by specifying global and/or individual node and coefficient precision values. *MDNAP* also offers control over quantization (round, floor, ceiling or truncate) and overflow (overflow, saturate or TC or unsigned binary overflow) methods.

5.1 Hardware Requirements

Scheduling information for sequential as well as parallel computation is provided, consisting of the branch or node computation sequence, as well as per-pixel processing time requirements. *MDNAP* can also calculate the memory requirements for the particular filter, taking into account the selected processing order.

VI. A Demonstration: Real-Time Color Video Acquisition and Display

MDNAP can be used to test SFGs on 3-D images, obtained using the real-time color video acquisition system. The system consists of a *SUN SPARCstation 2* configured with the *XVideo* X-Windows-based color graphics/ video card recently introduced by Parallax Graphics, Inc. Video-rate acquisition and display is achieved on this card by hardware JPEG compression and decompression using the JPEG chip from C-Cube Microsystems, Inc.

Images of any spatial resolution are supported by the XVideo card, up to a maximum of 640x480. At the default resolution of 320x240 and using an "acceptable" JPEG compression quality factor (i.e., one that results in little or no distinguishable image degradation), up to 2 minutes of live video (30 frames/sec) can be digitized. The maximum length is limited only by the amount of main memory and available disk space on the host *SPARCstation*. *MDNAP* has the added capability of directly reading and producing compatible JPEG-compressed images.

As an example, the filter in Figure 1(c) has been implemented using *MDNAP* and employed to filter a 100-frame (320×240) video sequence of a taxiing aircraft, filmed through a wire fence such that the plane appears stationary and such that the fence moves across the image with approximately uniform velocity. Visible blocking and noise have been added to the sequence as a result of compression with a poor image compression quality factor. A frame from this sequence is shown in Figure 3. The corresponding filtered frame is shown in Figure 4.

Figure 3. Original Sequence Frame

Figure 4. Filtered Frame

Clearly, *MDNAP* demonstrates that the filter successfully removes the moving fence and significantly reduces the noise and blocking effects of JPEG compression.

VII. References

[1] Dudgeon, D. E. and Mersereau, R. M., *Multidimensional Digital Signal Processing*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1984.

[2] L. T. Bruton and N. R. Bartley, "A general-purpose computer program for the design of two-dimensional recursive filters - 2DFil", *Circuits, Systems, and Signal Processing*, Vol. 3, No. 2, pp. 243-264, 1984.

[3] L. T. Bruton and N. R. Bartley, "Three-dimensional image processing using the concept of network resonance," *IEEE Trans. on Circuits and Systems*, Vol. CAS-32, No. 7, pp. 664-672, August 1985.

[4] A. A. Choudhury, L. T. Bruton, "Multidimensional Filtering Using Combined Discrete Fourier Transform and Linear Difference Equation Methods", *IEEE Trans. Circuits and Systems*, Vol. 37, No. 2, pp. 223-231, February 1990.

[5] R. P. Roesser, "A discrete state space model for linear image processing", *IEEE Trans. Automat. Contr.*, Vol. AC-20, pp. 1-10, Feb. 1975.

[6] T. C. Strecker, "Two-Dimensional Discrete Filters Using Spatial Integrators", *M.Sc. Thesis*, The University of Calgary, Calgary, Canada, 1983, Chapter 2.

[7] J. Foley and A. van Dam, "Computer Graphics Principles and Practice", Adison-Wesley, Reading, Mass., 1990, pp. 722-732.