Chapter 1 Appendix

Barriers to Learning: Science-speak, Numbers and Mental Models

1.1 Introduction

Although the impact of science and technology on modern life is clearly increasing, only a small proportion of the human population is trained in the field of science. For those whose expertise lies outside of science, there is often a certain mystery and strangeness associated with the worlds of science and technology. The very language of science is confusing to many and there is a sense that a specialized set of skills and knowledge is required in order to appreciate and understand the field. Very often, the curious and enquiring layperson is discouraged by the heavy use of mathematics and the description of phenomena that lies outside of the world of our everyday sensory experiences.

While there are undoubtedly many mental challenges when learning a new field, there are three particularly important aspects of science that often create initial barriers to effective learning. The first barrier is the specialized and sometimes confusing vocabulary that scientists use to describe objects and phenomena. The second barrier is the use of unimaginably large and small numbers to measure and quantify scientific phenomena and the third barrier is the increasing dependence of scientific explanations on *abstract* concepts or mental models that are apart from the physical world that we perceive via our senses. Accordingly, the next three sections address these three challenges that face newcomers to science.

1.2 Science-speak

Science-speak is the potentially confusing use of everyday words or phrases to describe scientific properties or concepts. In many cases, the scientific meaning of a word or phrase bears no relationship to its everyday meaning. For example, some of the properties of material objects within atoms (i.e. the so-called sub-atomic objects) have been arbitrarily assigned to particular words that have preexisting meanings outside of science, such as ‘spin’, ‘charge’, ‘color’, ‘charm’, ‘strangeness’ and ‘beauty’. Similarly, systems of physical objects behave in ways that have been assigned such words as ‘complex’, ‘chaotic’, ‘open’, ‘closed’, ‘dissipative’, ‘nonlinear’, ‘random’, ‘deterministic’ and so on. Numbers exist in categories that are described by words such 'real', imaginary', 'complex', 'irrational' or 'rational', 'even' or 'odd', 'prime' or 'non-prime', and so on. All of these words have everyday meanings that are not to be confused with their scientific meanings.

Some scientific words may have different meanings within science. For example, the word 'complex' has already been used twice in the previous paragraph: first as a property of a sub-atomic particle and second as a type of mathematical number. These two scientific meanings of the word 'complex' are completely different and represent a further level of potential confusion. As a final example of science-speak, the word 'field' in mathematics has a different meaning than the word 'field' in physics and neither meaning corresponds to the everyday meaning of 'an open land area free of woods and buildings'. In this book, the potential confusion of science-speak is minimized by carefully defining necessary scientific words at the place in the text where they are first employed or by defining them in the Glossary. The letter *g* in brackets after a word implies that it can be found in the Glossary.

The advice to newcomers to the field of science is always to search for the scientific definition of the word and never to assume that the everyday meaning is the same as with the scientific meaning. Be prepared to learn new meanings for scientific words that may have very different meanings outside of science.

1.3 Unimaginable and Unwieldy Numbers

The reader who does not wish to tackle the numerical aspects of this subject is advised to skip this sub-section and move on to the more important topic of mental images and mental models.

1.3.1 On the Prefixing of Measurement Units

The standard units of measurement for length, mass and time are metres, grams and seconds, respectively. However, the distance showing on the odometer of our automobiles is shown in *kilo*meters, rather than meters, because typical odometer distances in *meters* are too large and too detailed for everyday use. Similarly, the weight of a human body is measured in *kilo*grams, daily rainfall in *milli*meters and the clock rate of modern computers in *giga*-cycles per second (or giga-Hertz (GHz), where 1 Hertz is defined as 1 cycle per second. This prefixing of the standard units of measurement, using such prefixes such as *kilo-* , *milli-, giga-* and *mega-*, is universally employed in science.

The prefixes in the following table are commonly encountered:

pico-	*nano-*	*micro-*	*milli-*	*kilo-*	*mega-*	*giga-*
$\frac{1}{1,000,000,000,000}$	$\frac{1}{1,000,000,000}$	$\frac{1}{1,000,000}$	$\frac{1}{1,000}$	1,000	1,000,000	1,000,000,000

Microelectronics: Scientists in the field of microelectronics think in lengths of micrometers where, according to the table, 1 micrometer is one millionth (i.e. $1/1{,}000{,}000^{th}$) of a metre. The word 'micron' is often used instead of micrometer. Microelectronic devices, such as transistors, are capable of switching from one state to another (e.g. from the 'on' to the 'off' states) in a few picoseconds (psec) where 1 psec is one millionth of one millionth ($1/1{,}000{,}000{,}000{,}000^{th}$) of a second. Microelectronic circuit designers are comfortable working with lengths of microns and time periods picoseconds, in spite of the fact that both are unimaginably small in terms of the everyday objects that our senses perceive in the world around us. The length of each transistor on the silicon-based chips, that they carefully design, is typically about 1/5th of a micron.

Nanoscience and Nanotechnology: Molecular chemists need to imagine and understand how molecules come together in space and time during chemical reactions. Nanotechnology is about making materials and machines by designing and manufacturing the details at the molecular or atomic scales. Therefore, both molecular chemists and nanotechnologists think in terms of nanometers. It is convenient for them to imagine an inter-molecular distance of 5.7 nanometers, rather than 0.00000000057 meters.

1.4 Unimaginably Large and Small Numbers in Science

The following is a more extensive table of prefixes that are used to rescale units, covering 48 OMs

Source: http://en.wikipedia.org/wiki/SI_prefix

SI prefixes

10^n	Prefix	Symbol	Short scale	Decimal equivalent
10^{24}	yotta	Y	Septillion	1 000 000 000 000 000 000 000 000
10^{21}	zetta	Z	Sextillion	1 000 000 000 000 000 000 000
10^{18}	exa	E	Quintillion	1 000 000 000 000 000 000
10^{15}	peta	P	Quadrillion	1 000 000 000 000 000
10^{12}	tera	T	Trillion	1 000 000 000 000
10^{9}	giga	G	Billion	1 000 000 000
10^{6}	mega	M	Million	1 000 000

10^{3}	kilo	k	Thousand	1 000
10^{2}	hecto	h	Hundred	100
10^{1}	deca deka	da	Ten	10
10^{0}	*none*	*none*	One	1
10^{-1}	deci	d	Tenth	0.1
10^{-2}	centi	c	Hundredth	0.01
10^{-3}	milli	m	Thousandth	0.001
10^{-6}	micro	µ	Millionth	0.000 001
10^{-9}	nano	n	Billionth	0.000 000 001
10^{-12}	pico	p	Trillionth	0.000 000 000 001
10^{-15}	femto	f	Quadrillionth	0.000 000 000 000 001
10^{-18}	atto	a	Quintillionth	0.000 000 000 000 000 001
10^{-21}	zepto	z	Sextillionth	0.000 000 000 000 000 000 001
10^{-24}	yocto	y	Septillionth	0.000 000 000 000 000 000 000 001

1.4.1 On The Normalization of Quantities

We regularly read about national debts in *trillions* of dollars and human population in *billions* of people. Although we might know that a trillion is 1,000,000,000,000 and a billion is 1,000,000,000, this does not in itself convey the underlying meaning and implications of the national debt. One way to *understand the implication* of a national debt, of say 3 trillion (i.e. 3,000,000,000,000) dollars, is to express it on a per person basis. Suppose the national population is 150 million (150,000,000) people. Then, the national debt *per person (i.e. per capita)* is simply 3,000,000,000,000 dollars divided by 150,000,000 people which is $20,000 *per capita*. By *normalizing* the national debt to a per capita basis, it becomes meaningful for the human mind because $20,000 *is* a meaningful amount of money (in terms of what it will buy). Scientists normalize numbers for much the same reason: that is, to make quantities meaningful when they think about them. Two examples follow.

Normalizing Distance to Units of Light-Years: For example, the distance in metres from the planet Earth to the nearest group of stars, Alpha Centauri, is approximately $40,670,000,000,000,000$ meters, which is an unimaginably large and unwieldy number of meters. Astronomers normalize such enormous distances to the distance that light travel in one year, which is approximately $9,460,000,000,000,000$ meters per year. The distance $9,460,000,000,000,000$ meters is defined as one *light-year (ly)*. Normalizing the distance of Alpha Centauri to light-years, its distance from Earth is simply $(40,670,000,000,000,000) \div (9,460,000,000,000,000) \approx 4.3$ ly. The number 4.3 is not unwieldy and its magnitude is easily imagined. Physically, it takes 4.3 years for the starlight to reach us from Alpha Centauri. If we observe Alpha Centauri from Earth at this moment, we are seeing it as it was 4.3 years ago.

Normalizing Electronic Charge to Coulombs: Electrons carry a property known as negative charge. If we write this amount of negative charge as $-e$ then e is the magnitude of the *elementary charge* (as conveyed by one electron). It turns out that most practical measurements of the charge carried by electrons involve the total charge carried by many millions of millions (i.e. trillions) of electrons, where just 1 trillion electrons is the unimaginably large number 1,000,000,000,000. Clearly, using the elementary charge e leads to highly unwieldy measured values of total charge. Scientists have solved this problem by adopting the *normalized* unit of charge, known as the Coulomb (C), where one Coulomb is defined as the collective negative charge carried by

641,509,6291,526,500,000 electrons. A Coulomb *is* a practically convenient quantity of charge in many typical situations.

Electrical current is the rate at which charge crosses a surface and is therefore expressed in *Coulombs per second* and has units of Ampere (Amp), where 1 Amp is conveniently defined as 1 Coulomb per second. Typical electrical household appliances deliver electrical currents that do not exceed about 30 Amps, a convenient and imaginable number. Imagine how unwieldy it would be if we expressed a current of 30 Amps as '30 times 641,509,6291,526,500,000e' elementary charges per second.

1.4.2 Scientific Notation

Scientific notation is simply that: a way of writing down a number in the field of science. It has the benefit of allowing us to directly estimate the magnitude of an unwieldy number in terms of its so-called *order of magnitude* (OM).

First, we need to understand exponential notation for a number that is a multiple of 10. The exponential notation 10^{33} means '*the number 1 followed by 33 zeros* ' which is the same as '*the number 1 multiplied 33 times by the number 10*'. Therefore, 10^{33} is shorthand notation for the unwieldy large number

$$1,000,000,000,000,000,000,000,000,000,000,000 .$$

By convention, we speak '10^{33}' as "ten to the power of thirty three" where 33 is the so-called power of the number 10. With this in mind, here is an example of how to write the unwieldy 33 digit number

$$471436792003485961104633200053302$$

in scientific notation. We write this number as

$$0.471436792003485961104633200053302 \text{ x } 10^{33}$$

This notation is in two parts. Prior to the multiplication sign, we have the equally unwieldy 0.471436792003485961104633200053302 that is referred to as the *mantissa*. The second part is written as 10^{33} and is known as the exponent of the number. Note that you can recover the original 33 digit integer from the mantissa by moving the decimal point *to the right* by the number of decimal places given in the power of the exponent (33 in this case).

We can *estimate* the magnitude of a number by truncating its mantissa. For example, truncating the above mantissa to 2 decimal places yields 0.47 and the resultant estimate of the number in scientific notation is simply $0.47 \text{ x } 10^{33}$.

A number that has a magnitude that is much less than unity, such as the decimal number 0.00049, is written in scientific notation as $0.49 \text{ x } 10^{-3}$ (where 10^{-3} is shorthand notation for the fractional number that results from dividing the number 1 three times by 10, which is 1/1000.) The number 3 in the *negative* power of 10^{-3} is therefore the number of decimal places that the decimal point must be moved *to the left* in the mantissa (0.49) to yield the original decimal number 0.00049.

The interested reader may refer to Appendix ?.? to learn about the ease with which unwieldy numbers may be multiplied and divided when written in scientific notation.

1.4.3 Multiplying and Dividing Unwieldy Numbers Using Scientific Notation

Multiplication and division are especially easy using scientific notation. The rules are as follows. *When multiplying numbers in scientific notation, the mantissas are multiplied and the powers of the exponent are added*. For example, $(1.5 \text{ x } 10^{24})$ x $(0.5\text{x}10^{48}) = (1.5 \text{ x } 0.5) \text{ x } 10^{(24+48)} = 0.75 \text{ x } 10^{72}$ which is a very large number, relative to 1. *When dividing numbers in scientific notation, the mantissas are divided and the power of the exponent of the divisor is subtracted from the power in the numerator*. For example, $(1.5 \text{ x } 10^{24}) \div (0.5\text{x}10^{48}) = (1.5 \div 0.5) \text{ x } 10^{24-48} = 3.00 \text{ x } 10^{-24}$, which is a very small number relative to 1. If $3.00 \text{ x } 10^{-24}$ is a length in meters, then according to the table in Appendix A, it is more conveniently written as 3 yoctometers.

1.4.4 Order of Magnitude

The *order of magnitude* (OM) of a number is an approximate but often useful estimate of its magnitude and may be written directly from its scientific notation by arranging for the mantissa to lie in the range from approximately 0.33… to 3.33. *Then, the OM of the number is simply the power of the exponent.*

For example, consider the number $.008 \text{ x } 10^{33}$. We arrange for the mantissa to lie in the required range of 0.33… to 3.333 by adjusting the decimal point *two* places to the right in the mantissa to yield a new mantissa of 0.8, requiring that we now reduce the power of the exponent by two (from 33 to 31) so that the value of the number remains unaltered, yielding $0.8 \text{ x } 10^{31}$. Then, the OM of

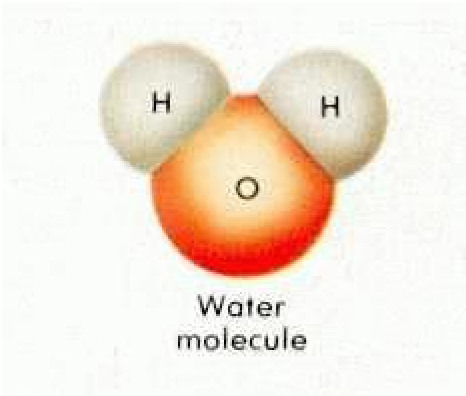
$$.008 \text{ x } 10^{33} = 0.8 \text{ x } 10^{31}$$

is 31.

When describing scientific phenomena, it is often sufficient to refer to a force, distance, speed, etc. or the *ratio* between two numbers in terms of its OM rather than in precise numerical terms. Creative thinking and conversations about science often involve the concept of OM. We shall very soon make use of the concept of OM.

1.5 Mental Images and Mental Models

A third challenge for lay readers and scientists alike is to be able to *imagine* objects and phenomena that have no counterparts among the objects and phenomena that we sense directly, during our everyday lives, via our biological senses (of which vision is the primary sense). For example, our senses are oblivious to objects as small as biological cells, much less to the molecules and atoms that they contain. Nor can we directly sense distant galaxies or the details of the stars or the myriad of objects in interstellar space. Although modern scientific instruments, such as microscopes and telescopes, allow our sense of vision to *indirectly* resolve such objects, we cannot directly sense or intuitively predict their behaviour on the basis of our experience of how objects behave in our immediate everyday environment. We are forced to describe many of the physical objects of science that lie beyond our senses by means of *mental models*. For this, we employ imagination, creativity and mathematics. For example, science widely employs mental models to represent molecules and the ways in which they interact, where spheres represent the locations of the constituent atoms and where interconnecting rods (or sticks) are used to represent the attractive forces that bond atoms to their neighbours. The mental model of a molecule of water is shown in the Figure. These models aid the human mind to imagine the relative spatial locations of atoms and the



inter-atomic forces within the molecule but they do not correspond to the ultimate physical reality, which is forever unknown to our senses and therefore to our minds.

Mathematical equations are very often used to mentally represent physical phenomena, such as the equations that describe the orbits of planets around the Sun or the equations that describe the ubiquitous electromagnetic waves that propagate through space, deliver energy to the Earth from the Sun and whose existence explains the structure of almost all material objects on Earth. Electromagnetic waves are conveyed from object to object by unimaginably small entities called photons and it makes no sense to ask what individual photons might 'look like' in terms of the HVS. The model of an electromagnetic wave is essentially mental and mathematical. Physicists will sometimes sketch photons, as shown in the Figure, in order to convey some of their characteristics, but their sketches do not describe the physical reality of the photon. The mental model explains some aspects of the behaviour of the photon.

As a final example of the mental representation of phenomena, we know that the Earth's surface consist of gigantic so-called tectonic plates that embrace the continents and that drift slowly over time, thereby creating the unimaginably large forces that move the land and shape our world, creating the mountains, valleys, earthquakes, volcanoes and eventually the conditions for Earthly life. Our senses are incapable of directly resolving this drift of the tectonic plates over time because it takes place too slowly, relative to the length of our lives. We therefore build mental models of the process of continental drift and we observe computer-generated visualizations, running millions of times faster than reality but fast enough for human minds to appreciate the dynamics of continental drift. A few seconds of computer-simulated continental drift might represent the physical reality of millions of years of drift.

Many of the mental models and processes that explain scientific phenomena are counterintuitive or mysterious. *This should not surprise us because there is no reason why our minds should have evolved over the generations with a ready-made capability to think intuitively about physical objects and physical behaviour that lie beyond our sensory range*.

We will later find that, as science has progressed over the last several thousand years, the need for mental models and abstraction has increased significantly. Today, almost all of modern science inherently employs abstract mental models as the basis for explaining phenomena in the physical world.