Three-Dimensional Linear Trajectory Filtering Using the DWT and the Mixed-Domain Approach

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***Abstract* - A method is proposed which combines the two-dimensional (2-D) orthogonal discrete wavelet transform (DWT) and mixed-domain (Mixed-D) filtering to selectively enhance or attenuate a 2-D signal moving along a linear trajectory at a constant velocity. Such signals are common in video coding applications. The 2-D DWT is applied separately to each frame of the three-dimensional (3-D) input sequence, resulting in a set of multiresolution (subband) image sequences. Mixed-D filtering is then used to enhance or reject linear trajectory signals within each subband image sequence. The output for each frame is formed by inverting the wavelet transform using the Mixed-D processed sequences. This method of filtering lends itself to use in subband video coding systems, which are becoming increasingly popular, and permits selective Mixed-D filtering of the subband signals depending upon the energy content within the subbands. In this contribution, we present an overview of the proposed algorithm and present preliminary results which compare favourably in terms of arithmetic complexity, storage requirements, and the subjective quality of output, to those obtained using the Mixed-D technique.**

I. Introduction

We consider the important class of signals in video processing that correspond to a two-dimensional (2-D) object moving with approximately constant velocity along a linear trajectory. It can be shown that the spectral energy of such a signal lies approximately within a three-dimensional (3-D) plane having a normal defined by the direction of motion [4]. These *linear trajectory* (LT) signals can be selectively enhanced or attenuated using 3-D filters having a passband (or stopband) that is coincident with the signal plane [4].

It has been shown that it is possible to combine the discrete Fourier transform (DFT) and linear difference equation (LDE) filtering methods in order to filter multidimensional (m-D) signals [6]. This approach, referred to here as mixed-domain (Mixed-D) processing, can also be used to filter LT signals [5]. Consider a 3-D LT input signal $x(n_1, n_2, n_3)$, where n_1, $1 \le n_1 \le N_1$, and n_2, $1 \le n_2 \le N_2$, are spatial indices, and n_3 is a temporal index. In the Mixed-D technique, it has been proposed that the 2-D DFT be applied to each frame, $x(\bullet, \bullet, n_3)$, yielding a transformed image $X(k_1, k_2, n_3)$, $1 \le k_1 \le N_1$, $1 \le k_2 \le N_2$ [6]. Each of the resulting complex 2-tuple frequency points within the transformed frame is then filtered using a complex-coefficient LDE to yield $Y(k_1, k_2, n_3) = X(k_1, k_2, n + n_3) * H(k_1, k_2, n)$, where $H(k_1, k_2, n)$ is the impulse response of the LDE, and * represents convolution over the variable n. The set of LDE filters, $H(k_1, k_2, \bullet)$, $1 \le k_1 \le N_1, 1 \le k_2 \le N_2$, is selected to attenuate or transmit energy coincident with the signal plane of the LT object. The output frame, $y(\bullet, \bullet, n_3)$, is computed by taking the 2-D inverse DFT of $Y(\bullet, \bullet, n_3)$.

A limitation to the proposed Mixed-D algorithm [6] is that each 2-D frequency 2-tuple, (k_1, k_2), has associated with it, in general, a unique LDE filter. Assuming for simplicity that N_1 and N_2 are even, a real valued input signal $x(n_1, n_2, n_3)$ requires a total of $N_{LDE} = (N_1 N_2/2) + N_1/2 + N_2/2$ unique LDE filters. For a complex-valued input $x(n_1, n_2, n_3)$, $N_{LDE} = N_1 N_2$. For large N_1, N_2, the amount of memory required to store the LDE coefficients may make Mixed-D filtering prohibitive, which is often the case in real time video processing. Given *a priori* knowledge of the 2-D magnitude spectrum of the input signal, however, the arithmetic complexity may be reduced by neglecting some of the LDE filtering operations.

In this contribution, we present preliminary results which show that the 2-D DWT may be used to partition the spectrum of the input LT signal into subbands, which then may be selectively Mixed-D filtered. Results obtained using this approach indicate that it is possible to reduce the number of LDE filters required for Mixed-D filtering, without significant loss of subjective image quality. This approach further lends itself to recently proposed video coding techniques that make use of the wavelet transform [7][8][9].

II. Combined DWT/Mixed-D Filtering of Linear Trajectory Signals

The wavelet transform has recently emerged as an efficient tool for processing signals within a multiresolution framework [1][2]. For discrete-time (and therefore resolution-limited) one-dimensional (1-D) signals, a tree-structured perfect reconstruction (PR) filter bank can be used to realize an orthogonal wavelet decomposition. In order to implement a 2-D DWT, we use the separable extension of the 1-D decomposition. Therefore, the 2-D DWT of the frame $x(\bullet, \bullet, n_3)$ computed to J multiresolution levels and using a 1-D dilation parameter, M [2], consists of $R = J(M^2 - 1)$ difference subband image (subimages) and one low resolution subimage. In this contribution we use dyadic wavelets, in which case $M = 2$. We denote the set of subimages at frame n_3 by $W^r x(\bullet, \bullet, n_3)$, where $0 \le r \le R$. The low resolution subimage ($r = 0$) is of size $(N_1/2^J) \times (N_2/2^J)$, while, for each multiresolution level j, $1 \le j \le J$, there are three difference subimages, each of size $(N_1/2^j) \times (N_2/2^j)$.

The Combined DWT/Mixed-D filtering algorithm consists of computing the 2-D DWT of each input image frame and then Mixed-D filtering the resulting subband sequences, $W^r x(\bullet, \bullet, n_3)$. The output is formed by computing the inverse 2-D DWT of Mixed-D filtered sub-sequences. A block diagram of Combined 2-D DWT/Mixed-D filtering method is shown in Figure 1.

Note that $R + 1$ sets of LDE filters are required when Mixed-D filtering, each specific to a resolution and subimage sequence [10]. It can be shown that for PR filters having an ideal frequency response, there is no change in the LDE filters required to perform Mixed-D filtering in the subbands [10]. What does change, however, is the corresponding frequency associated with each LDE filter. This is due to the frequency shifting which occurs during the subsampling operation of the 2-D DWT. This result suggests that, in practice, PR filters with good pass-band and stop-band frequency response be used in order to leave the LDE filter design step unchanged in Combined DWT/Mixed-D filtering.

Consider the situation where Combined DWT/Mixed-D filtering is used to filter an input signal containing a LT signal with velocity $v_0 = (c, r)$, where c and r are the horizontal and vertical components of the velocity (in pixels/frame), respectively. After computing the 2-D DWT to J multiresolution levels, the velocity of the LT object within each subband signal at the j^{th} multiresolution level (assuming inter-pixel distances are constant across resolutions) is given by

$$v_j = \left(\frac{c}{2^j}, \frac{r}{2^j}\right). \tag{1}$$

Thus, the LDE filters associated with each subband would be designed to filter LT objects at a reduced velocity, given by (1), keeping in mind the frequency shifting caused by the 2-D DWT. As stated above, this does not change the overall number or the design of the LDE filters, assuming that ideal PR filters are used.

With the spectrum of each frame of the input signal partitioned via the 2-D DWT, however, it is possible to selectively Mixed-D filter the various subband signals. Since we expect most objects to have their spectral energy concentrated in the low pass region, it is possible to neglect filtering the high pass (detail) signals or to Mixed-D filter the high pass signals only if they contain sufficient signal energy. Another option afforded by combined DWT/Mixed-D filtering is to filter *all* subbands using the *same* set of LDE filters. This strategy may be used to reduce the complexity of the overall implementation by using one Mixed-D processing element to filter (on a time-shared basis) all subband signals at a given resolution.

III. Results

To test the Combined DWT/Mixed-D filtering algorithm, we have used an available 9 second (270 frame) 320×240 JPEG compressed ($Q = 100$) video sequence of a road containing moving vehicles [11]. The previously described input signal, $x(n_1, n_2, n_3)$, has been obtained from the video sequence by extracting a 202×220 portion of each decompressed frame, and interpolating (using bilinear interpolation) to frames of size $N_1 = 256$, $N_2 = 256$. Frame 220 of the input, $x(\bullet, \bullet, 220)$, is shown in Figure 2 (a). The velocity of the cement truck in $x(n_1, n_2, n_3)$ was estimated to be $v_0 = (-2.5, 0.27)$ pixels per frame.

For comparative purposes, the sequence has been Mixed-D filtered as described in [5]. Frame 220 of the Mixed-D filtered output is shown in Figure 2 (b). Since the input signal is real valued, 33024 unique LDE filters have been used during Mixed-D filtering.

The 2-D separable DWT of the combined filtering method has been computed using twenty-tap (i.e. FIR) dyadic 1-D PR filters, which have been designed using the Type 2 method described in [3]. The 1-D low pass filter, $H_0(z)$, contains four zeros at $z = -1$, and thus satisfies the 1-D regularity requirement [2]. In addition, $H_0(z)$ contains six more zeros on the unit circle (three complex-conjugate pairs) which are used to provide improved stop band response. The remaining zeros of $H_0(z)$ have been selected so that the pair of filters satisfy the PR property. In order to handle finite length data effects, 2-D filtering has been performed assuming that all images are periodically extended.

The Combined DWT/Mixed-D method has been used to enhance LT objects at velocity v_0 in $x(n_1, n_2, n_3)$. The 2-D DWT has been computed to $J = 1$ multiresolution level and all subband signals have been filtered using the same set of second order LDE filters as used in Mixed-D filtering. (*No compensation for the phase response of the 2-D DWT was used during LDE filter design, since the Mixed-D filtering step is sensitive only to the motion, and not the position, of the LT object.*) Note that this approach requires the same number of unique LDE filters as in Mixed-D filtering, but requires approximately four times the number of LDE filtering related operations [10].

The result of combined filtering for frame 220 is shown in Figure 2 (c), and clearly demonstrates the enhancement of the LT object. Comparing the Mixed-D filtered frame with the Combined DWT/Mixed-D filtered frame, illustrates that the combined method has slightly better high frequency response, as indicated by the object's increased detail. Of course, this additional detail is at the price of an increasing number of arithmetic operations. Similar results to those shown for Frame 220 have been obtained throughout the entire sequence of 270 frames (after accounting for start-up transients).

The Combined DWT/Mixed-D method has also been used to filter $x(n_1, n_2, n_3)$, *but with the Mixed-D processing step omitted for all difference (i.e. high pass) subband signals.* (During computation of the inverse DWT step, zero valued difference signals have been used.) Effectively, therefore, only the low resolution image ($r = 0$) sequence, of size $128 \times 128 \times 300$, has been Mixed-D processed. Consequently, the number of unique LDE filters required is 8256. The output at frame 220, shown in Figure 2 (d), demonstrates the enhancement of the LT object, and shows little difference in subjective quality from that of the Mixed-D filtered frame. In fact, the average PSNR value between this sequence and the Mixed-D filtered series is over 40 dB for the entire 270 frame sequence [10]. *Relative to the number of LDE operations used in Mixed-D filtering*, this latter form of Combined DWT/Mixed-D filtering requires 57792 (35%) fewer complex storage locations, 57792 (35%) fewer complex multiplications, and 74304 (56%) fewer complex additions per frame [10].

IV. Conclusion

In this paper, we propose combining the 2-D discrete wavelet transform and mixed-domain filtering methods to filter linear trajectory signals. We have provided an example of the proposed method, and shown that it is possible, in some cases, to obtain results of comparable subjective quality to those of the Mixed-D technique using fewer distinct LDE filters and with reduced overall arithmetic complexity. Future work in this area could involve the use of an adaptive algorithm to determine when to filter the various subband signals.

V. References

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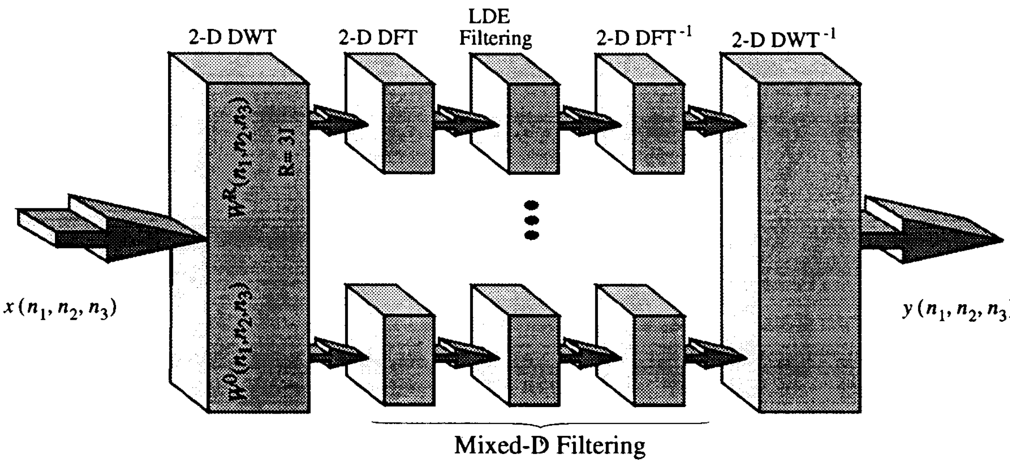

Figure 1 - Block Diagram of Combined DWT/Mixed-D filtering

Figure 2 - Results at Frame 220

(a) Original Input (b) Mixed-D filtered

(c) Combined DWT/Mixed-D filtered: All subbands filtered using the same LDE filters

(d) Combined DWT/Mixed-D filtered: High Pass Subband Signals Cleared during the Mixed-D filtering